Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713926090

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To cite this Article Lam, L.(1993) 'Reentrant solitons', Liquid Crystals, 14: 6, 1873 — 1876
To link to this Article: DOI: 10.1080/02678299308027722
URL: http://dx.doi.org/10.1080/02678299308027722

LIQUID CRYSTALS, 1993, VOL. 14, NO. 6, 1873–1876

Reentrant solitons

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Reentrant solitons are predicted to exist in homeotropic nematic liquid crystals under simple shear or in Poiseuille flow. The director equation of motion with the boundary effects due to the cell walls fully incorporated is given, which is then approximated and reduced to an effective equation of one spatial dimension with only two dimensionless parameters β and γ. For β fixed within a narrow range, as γ is varied a sequence of soliton–no soliton–soliton is obtained. This new reentrant soliton phenomenon is accessible to experimental verification.

1. Introduction

Reentrant phenomena exist in quite a number of materials in thermal equilibrium. These materials include Rochelle salt, magnetic superconductors, a mixture of nicotine and water, and liquid crystals [1, 2]. On the other hand, there are fewer cases of reentrant behaviour known to exist in non-equilibrium systems. Examples in non-equilibrium liquid crystals are the reentrant sequence of viscous fingers [3], and the stabilization of a moving nematic–isotropic interface in directional liquid-crystallization as evidenced by the bifurcation curve in the velocity–temperature gradient plane [4].

In this paper, reentrant solitons are theoretically shown to exist in homeotropic nematic cells under simple shear (SS) or in Poiseuille flow (PF), which obviously is a non-equilibrium system. As it turns out, these reentrant solitons appear only when the boundary effects of the finite cell thickness is taken into account. This brings us to the second motivation behind this investigation.

Solitons are involved in the mechanical, hydrodynamical, and phase transition properties of liquid crystals. They are important as a basic non-linear wave phenomenon, sometimes an influencing factor in the material properties, and in the switching mechanism of ferroelectric smectics [5]. Ten years ago, in the study of propagating solitons in nematics under simple steady shear [6], the boundary effects of the two cell surfaces were ignored completely. This procedure enabled one to reduce the problem to one dimension; the director equation of motion could then be analysed and analytic soliton solutions found in the high velocity regime [6, 7]. Even though good agreement with experiments is obtained, this theoretical piece of work leaves some questions unanswered. For example, (i) what is the difference between a homeotropic cell and a planar cell containing a nematic under shear? (ii) How thick does a cell have to be before the boundary effects can be safely ignored? In short, the boundary effect of the cell must be studied [7].

In the following, the director equation of motion of a nematic under either SS or PF, with the boundary effects included, is presented and analysed. Reentrant solitons (in a two parameter plane) are found; verifying experiments are proposed. Other ramifications of the boundary effects are discussed.

2. Reentrant solitons

Let θ be the angle between the director and the normal of a nematic cell of finite thickness d. The x and y axes are along the cell length and the cell normal, respectively (see figure 1). When the rotational moment of inertia is ignored and the one-elastic-constant approximation is used, the director equation of motion of a nematic under either SS or PF is given

$$K\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)-\gamma_1\frac{\partial\theta}{\partial t}+\frac{s}{2}(\gamma_1-\gamma_2\cos 2\theta)=0, \tag{1}$$

where $\theta=\theta(x,y,t)$ and s is the shear; $s=\text{const}$ for SS, and $s=s(y)$ for PF. For a homeotropic cell, the boundary condition is given by $\theta=0$ at $y=\pm d/2$.

Equation (1) cannot be solved analytically. To proceed further, one assumes

$$\theta(x,y,t)=f(y)\theta_m(x,t) \tag{2}$$

and

$$f(y)=\begin{cases}\cos(\pi y/d) & \text{for SS,}\\ g(y)/g(-y_m) & \text{for PF,}\end{cases} \tag{3}$$

where $g(y)\equiv 2y/d-\sin(\pi y/d)$; $y_m=(d/\pi)\cos^{-1}(2/\pi)\approx 0{\cdot}28\text{d}$, defined by θ_m being the maximum across the y direction at $-y_m$ (see figure 1 (*b*)). When equations (2) and (3) are substituted into equation (1), due to the approximate nature of equation (2), one is left with an equation which is not strictly valid for all y [7]. However, an effective equation of motion with the correct physics preserved may be obtained by setting $y=0$ (for SS) or $y=-y_m$ (for PF) in this equation. In the dimensionless form, this effective equation of motion is given by

$$\frac{\partial\theta_m}{\partial T}=\frac{\partial^2\theta_m}{\partial X^2}-2\beta\theta_m+\gamma+\cos 2\theta_m, \tag{4}$$

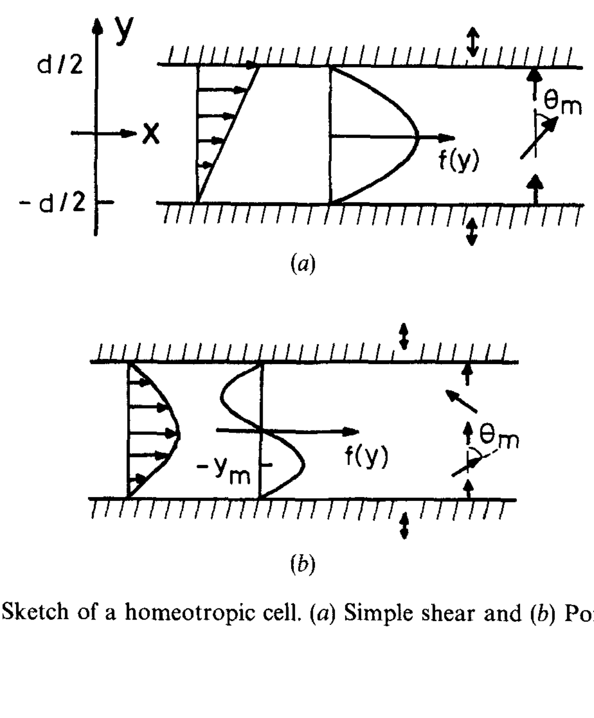

Figure 1. Sketch of a homeotropic cell. (*a*) Simple shear and (*b*) Poiseuille flow.

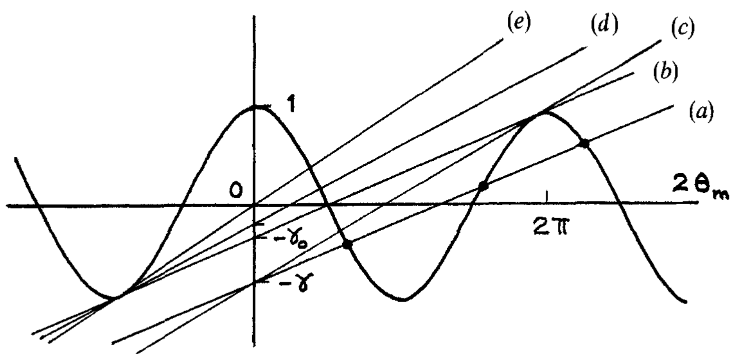

Figure 2. Graphical solutions of equation (5). Lines (*a*), (*b*), (*c*), (*d*) and (*e*) have slopes β, β_0, β_c (for $\gamma > \gamma_0$), β_c (for $\gamma < \gamma_0$) and β_1, respectively.

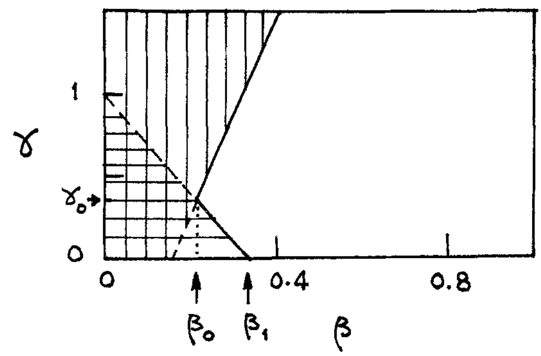

Figure 3. Soliton phase diagram in the (β, γ) plane. The shaded (blank) area on the left (right) of the β_c curve represents the soliton (no soliton) region. In the vertically (horizontally) shaded area, the solitons do not (may) contain vertical molecules with $\theta_m = 0$.

where $T \equiv t/\tau$, $\tau \equiv 2\gamma/s_m$, $X \equiv x/\lambda$, $\lambda \equiv [2K/(s_m|\gamma_2|)]^{1/2}$, $\gamma \equiv \gamma_1/|\gamma_2|$, $\beta \equiv KB\pi^2/(s_m d^2|\gamma_2|)$. γ_2 is assumed to be negative, as is the case for MBBA; $s_m \equiv s(-y_m)$ for PF, and $s_m = s$ for SS. $B = 1$ for SS; $B = -(d/\pi)^2 f''(-y_m) \approx 3{\cdot}66$ for PF. The only two (dimensionless) parameters in equation (4) are γ and β; both are positive. While γ is purely a material parameter, β is related to the cell thickness and the shear strength. A 'thick' cell where the boundary effects can be neglected is one for which $\beta \ll 1$. Note that this may be the case even if d is not too small; it is the largeness of the product $s_m d^2$ that counts.

The steady uniform states of the shearing nematic are given by

$$\cos 2\theta_m - (-\gamma + 2\beta\theta_m) = 0. \tag{5}$$

The solutions of equation (5) are shown as the intersecting points between the straight line (*a*) and the $\cos 2\theta_m$ curve in figure 2. Solitons exist whenever there are two or more such steady states [8]. Consequently, by inspecting figure 2 it is easy to see that for a given γ no soliton can exist if $\beta > \beta_c$, and solitons exist if $\beta \leqslant \beta_c$. Note that $\beta_c = \beta_c(\gamma)$, which is given by the slope of curve (*c*) for $\gamma > \gamma_0$, and that of curve (*d*) for $\gamma < \gamma_0$. Here the straight line with $\gamma = \gamma_0$ (and slope β_0) is shown as curve (*b*) in figure 2.

Numerical solutions for $\beta_c(\gamma)$ are plotted in figure 3, where $\gamma_0 \approx 0{\cdot}34$, $\beta_0 \approx 0{\cdot}22$ and $\beta_1 \approx 0{\cdot}34$. For β fixed and $\beta_0 < \beta < \beta_1$, as γ is varied one obtains the sequence soliton–no soliton–soliton, i.e. a reentrant soliton phenomenon. The existence of the reentrant solitons can be understood by looking at curve (*a*) in figure 2. By holding β fixed between β_0 and β_1 we see that the straight line cuts the $\cos 2\theta_m$ curve at only one point for a limited range of γ if γ is varied, i.e. there is no soliton within this range of γ.

In the case of SS, $s_m = u/d$ where u is the relative velocity of the two glass plates. One may observe the reentrant solitons experimentally by fixing u and varying d to tune β. For $K \sim 10^{-6}$ dyn, $|\gamma_2| \sim 1$ P, and $u \sim 1\,\mathrm{mm\,s^{-1}}$, the reentrant region occurs when d satifies $4{\cdot}5\,\mu\mathrm{m} > d > 2{\cdot}9\,\mu\mathrm{m}$. Similarly, for PF, assuming $s_m \sim u/(d/3)$ with u being the maximum flow velocity, the reentrant region occurs when $50\,\mu\mathrm{m} > d > 32\,\mu\mathrm{m}$. Alternatively, one may perform the experiment by fixing d and varying u to tune β. γ may be tuned by varying the temperature or using different materials. The estimates above show that these kind of experiments are easily accessible.

Note that the mere existence of SS or PF is insufficient for solitons to be observed. Some method of generating them is needed. See [7, 8] for more details.

3. Discussions

In the MBBA soliton experiments as analysed in [6], one has a PF with $s_m = (2{\cdot}25\,\mathrm{mm\,s^{-1}})/(15\,\mu\mathrm{m}) = 150\,\mathrm{s^{-1}}$, and $d = 50\,\mu\mathrm{m}$. Assuming $K = 6 \times 10^{-7}$ dyn and $\gamma_2 = -0{\cdot}8$ P, the corresponding β is approximately equal to $7{\cdot}23 \times 10^{-3}$. This extremely small β implies that the boundary effects of the cell can be safely ignored as in [6]. Also, from the result of § 2, the dimensionless soliton velocity η is still given by $\eta = c\tau/\lambda$, irrespective of the cell thickness, where c is the observed velocity of the soliton (as exemplified by a dark line under white light in this case) in the laboratory. Therefore, one still has $\eta \sim 10^3$ as estimated in [6] and the interpretation of the dark lines as A solitons moving with velocities much larger than the minimum velocity (the one predicted by marginal stability) is correct. See §§ 2.6 and 3.2.9 in [5] for more discussions.

In the case of a planar cell, an effective director equation of motion, following the same kind of approximation as in § 2, can be derived [7]. It differs from equation (4) only by the sign of γ, with θ_m now representing $\pi/2 - \theta_m$. One does not have reentrant solitons in this case [7]. See [7] for further discussions on the differences between a homeotropic and a planar cell when the nematic is under simple shear or for Poiseuille flow.

Finally, it should be pointed out that in the reentrant region of $\beta_0 < \beta < \beta_1$, the soliton at low γ is not identical to the reentrant soliton at high γ. As can be seen from figure 3, the former (latter) lies in the horizontally (vertically) shaded region. They are therefore different from each other in details even though both are indeed solitons.

This work is supported by a Syntex Corporation grant from the Research Corporation.

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